A CONTINUOUS METHOD TO QUANTIFY STRESS-STRAIN BEHAVIOR OF BIOLOGIC MATERIALS

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Introduction

Stress-strain curve of soft biological tissues has:
- a nonlinear (exponential) toe region.
- a linear region. [1]

Fitting a curve to this piecewise-but-continuous behavior is challenging.

Purpose

To apply a previously developed modeling method [1] while fitting the toe and linear regions simultaneously to ensures the continuity of the stress-strain curve and its derivative.

Methods

A continuous curve-fitting method was:
- implemented in MATLAB.
- evaluated with three data sets.

Continuous Method *(New!)*

- Simultaneously optimized both regions.
- Transition point *(p,q)* between regions was a variable in the optimization.
- *A*, *B*, and elastic modulus, *E*, were variables
- Minimized mean square error (*MSE*).

$$\sigma = \begin{cases} A\left(e^{B\varepsilon} - 1\right) & \forall \varepsilon \le p \\ E(\varepsilon - p) + q & \forall \varepsilon > p \end{cases}$$

Piecewise Method *(Traditional)*

- Linear region least-squares fit.
- Transition Point *(p,q)* at $R^2<0.99$. [2]
- Toe region fit to an exponential curve. [3]

Evaluation

- **Ideal Data** created using known parameters.
- **Noisy Data** created by adding zero-mean Gaussian random noise to Ideal Data.
- **Measured Data** collected from tensile tests of porcine lateral meniscus. [4]

Results

Ideal Data

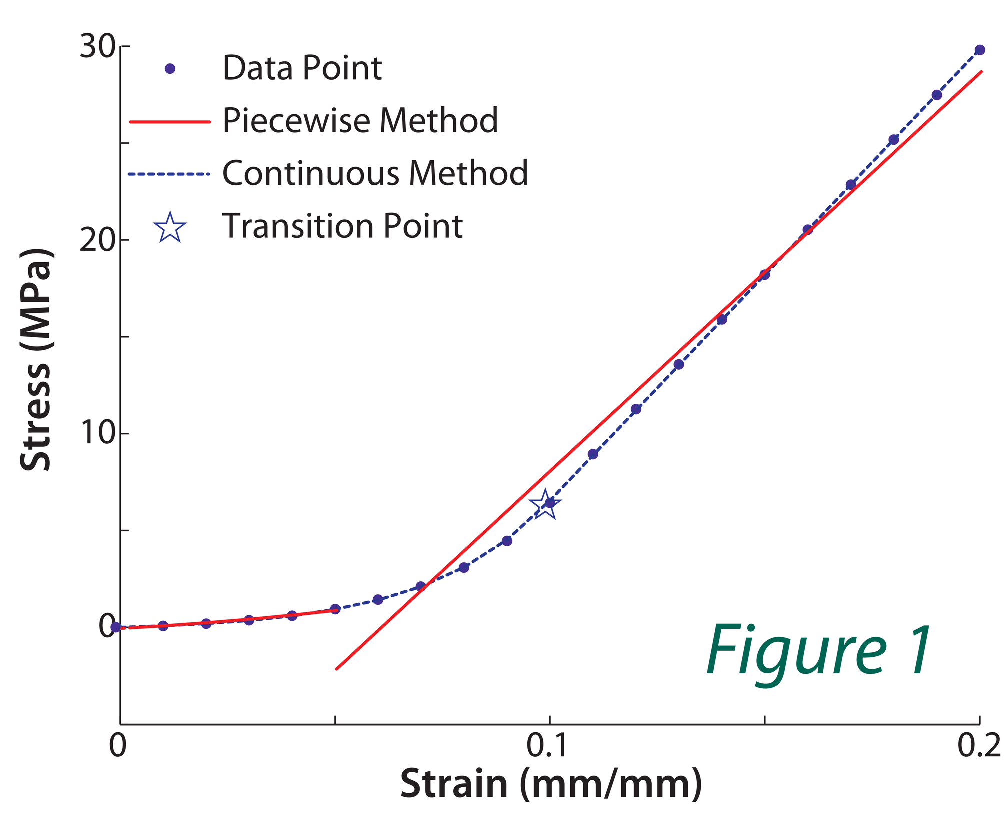

Figure 1

Table 1 Computed Value (Percent Error)

	A (MPa)	B	E (MPa)	p	q (MPa)	MSE (MPa)
Known	0.2	35	232	0.1	6.42	n/a
Continuous Method	0.186 (7%)	35.9 (2.6%)	233 (-0.4%)	0.099 (1.1%)	6.3 (1.9%)	0.0015
Piecewise Method	0.947 (374%)	13.7 (61%)	206 (11%)	0.05 (50%)	-0.585 (109%)	1.09

Continuous Method
- No discontinuity at Transition Point (Figure 1)
- Error at Transition Point (*p*,*q*): 1.10% strain, 1.87% stress
- Parameter Errors: Table 1

Piecewise Method
- Discontinuity at (*p*,*q*): 3.03 MPa (Figure 1)
- Error at Transition Point (*p*,*q*): 50% strain, 109% stress
- Parameter Errors: Table 1

Ideal Data with Noise

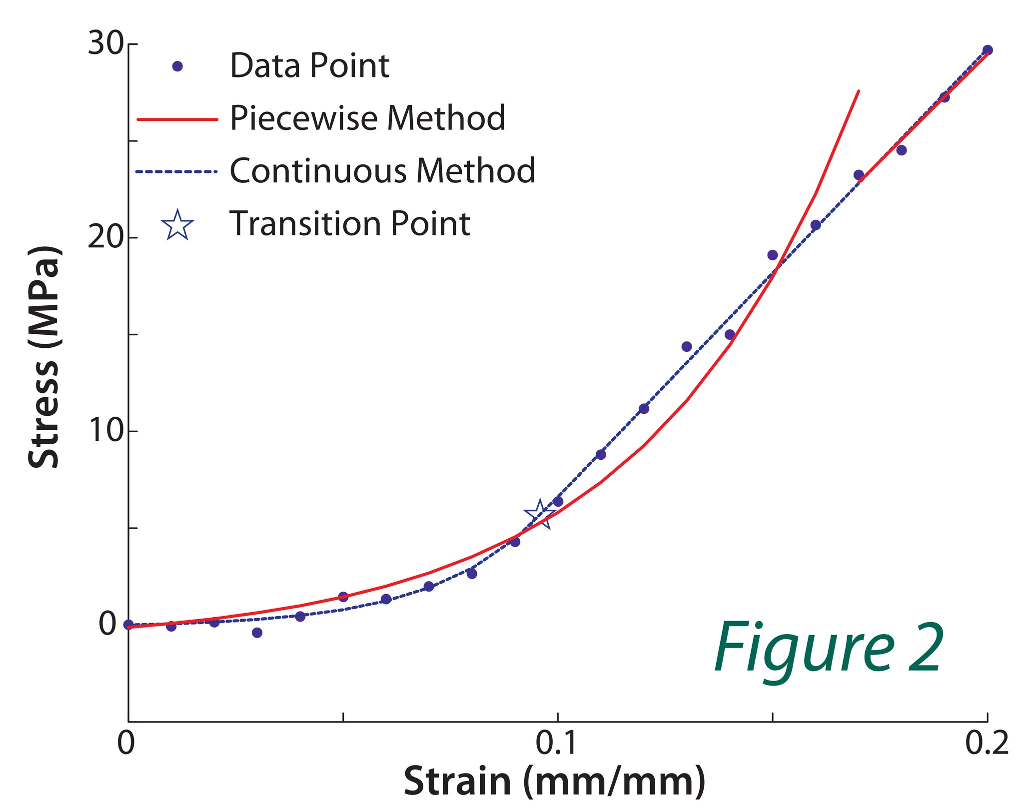

Figure 2

Table 2 Computed Value (Percent Error)

	A (MPa)	B	E (MPa)	p	q (MPa)	MSE (MPa)
Known	0.2	35	232	0.1	6.42	n/a
Continuous Method	0.123 (38.5%)	40.1 (14.6%)	232 (<0.01%)	0.096 (4.2%)	5.65 (12.0%)	0.194
Piecewise Method	0.878 (339%)	20.5 (41.4%)	221 (4.74%)	0.17 (70%)	25.2 (293%)	1.84

Continuous Method
- No discontinuity at Transition Point (Figure 2)
- Error at Transition Point (*p*,*q*): 4.2% strain, 12% stress
- Parameter Errors: Table 2

Piecewise Method
- Discontinuity at (*p*,*q*): 4.71 MPa (Figure 2)
- Error at Transition Point (*p*,*q*): 70% strain, 293% stress
- Parameter Errors: Table 2

Measured Data

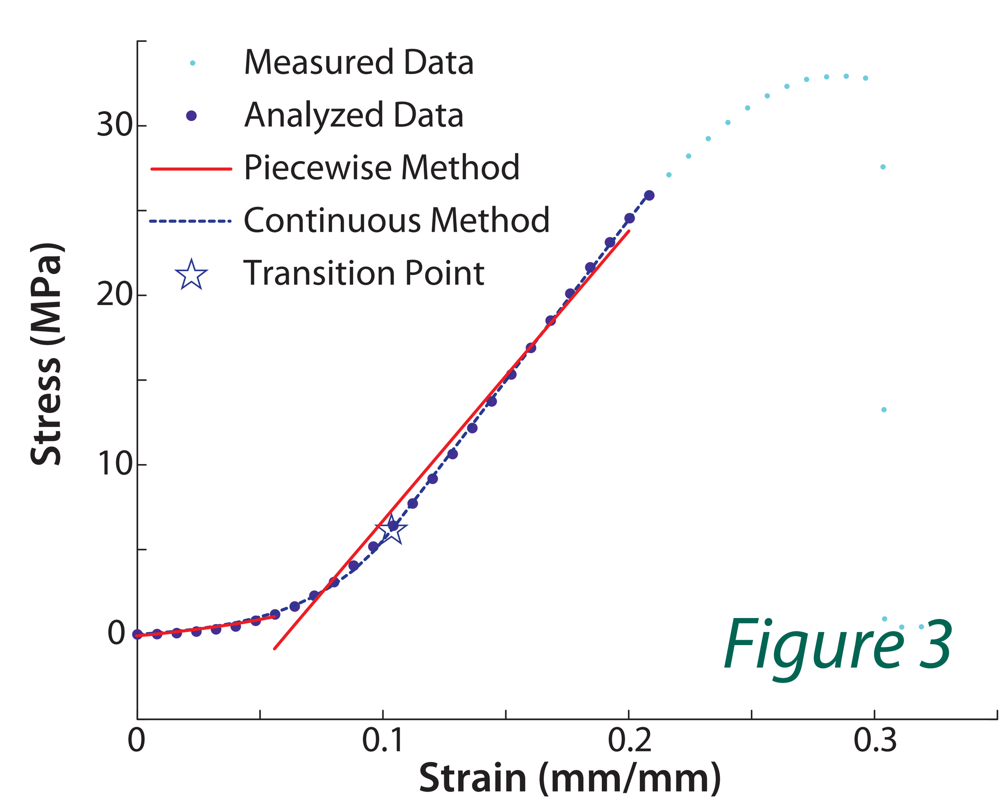

Figure 3

Table 3

	A (MPa)	B	E (MPa)	p	q (MPa)	MSE (MPa)
Known	n/a	n/a	n/a	n/a	n/a	n/a
Continuous Method	0.882	14.8	171	0.0561	0.0419	0.021
Piecewise Method	0.307	29.4	190	0.103	6.14	0.46

Continuous Method
- No discontinuity at Transition Point (Figure 3)
- MSE twenty times lower than Piecewise Method (Table 3)

Piecewise Method
- Discontinuity at (*p*,*q*): 1.9 MPa (Figure 3)

Discussion and Conclusion

The Continuous Method eliminates discontinuities in both the stress-strain curve and the modulus that are present in the traditional Piecewise Method.

Errors in the computed material parameters *A*, *B*, *E*, *p*, and *q* are dramatically reduced.

By reducing these errors, those using these parameters for finite element analysis, or as design goals for tissue engineered materials, will have more accurate results.

References

[1] Fung, Y.C. , 1967, Am J Physiol, 213(6), pp. 1532-1544.
[2] Voycheck, C. A. , *et al.* 2008, ASME SBC, Marco Island, FL.
[3] Woo, S. L. 1982, Biorheology, 19(3), pp. 385-396.
[4] Stabile, K. J. *et al.* 2010, Arthroscopy, 26(7), pp. 936-948.

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