Proceedings of the ASME 2010 Summer Bioengineering Conference (SBC2010)
June 16-19, Grande Beach Resort, Naples Florida, USA

SBC2010-19414

EMPLOYING TOPOLOGICAL EQUIVALENCE IN BIOMECHANICS

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INTRODUCTION

By its very nature, biomedical engineering is interdisciplinary, often combining tools and techniques originally developed for engineered systems and applying them to biological problems. Often techniques developed to solve one problem may be used to solve a very different problem. One useful concept from the field of mathematics is topological equivalence. Two dynamical systems are "topologically equivalent" if the phase portraits are qualitatively similar. That is, one portrait can be obtained from the other by a continuous transformation (Figure 1).

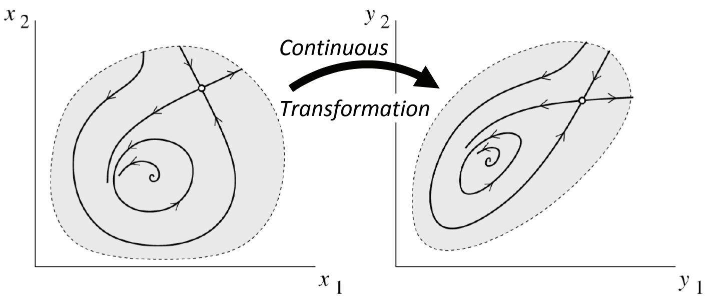

Figure 1: For these topological equivalence systems, the phase portrait of **y** can be created through a continuous deformation of **x**.

In dynamics, these seemingly different systems may be fundamentally equivalent and can even share the same equations of motion. In this abstract, we will show how topological equivalence can be used in solving the challenging problem of finding stable controller gain parameters for a mathematical model of a person balancing on an unstable seat (Figure 2).

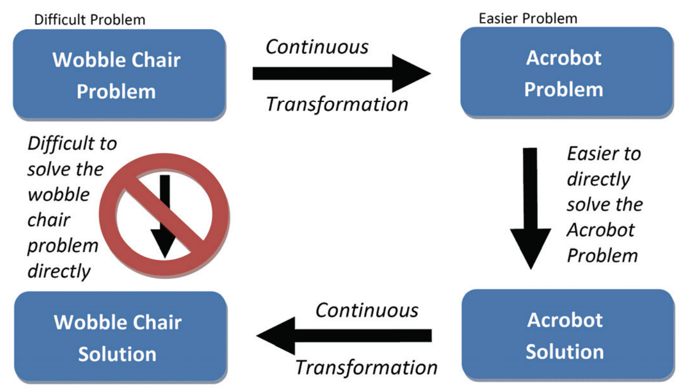

Figure 2: Flow map showing how topological equivalence is used to solve a difficult problem. A problem that is difficult to solve directly is transformed to an easier problem which is solved, then transformed back to obtain a solution to the original problem.

METHODS

The wobble chair (Figure 3) is an unstable seat apparatus used to evaluate spinal stability. In order to better understand the dynamics of the system, a mathematical model was developed [1]. Our goal was to perform forward dynamic simulations to study the stability of the torso using this model. However, we were unable to find adequate controller gain parameters for the PD controller that stabilized this inherently unstable system. As a result, we were having difficulty solving this problem directly.

Our approach to solve this difficult problem was to use the knowledge that the wobble chair was topologically equivalent to a known dynamic system, the Acrobot [2] (Figure 3). In fact, the only differences were the segment lengths, masses, and moments of inertia (constants). The differential equations controlling motion were identical.

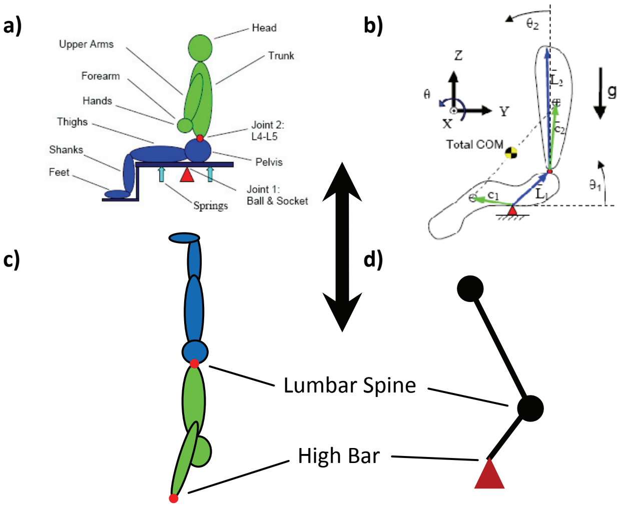

Figure 3: The wobble chair (a) and its mathematical model (b) [1]. The Acrobot (c) and its mathematical model (d).

RESULTS

After transforming the wobble chair model into the Acrobot, proportional and derivative control parameters were quickly found that produced stable system behavior, G_p = 9x10^3 Nm/rad and G_d = 6 Nm/rad/s. The controller was able to drive the combined center of mass near the vertical equilibrium position (Figure 4a).

Beginning with the solved Acrobot problem, the system parameters were slowly changed and the controller gains adjusted so that stability could be maintained during each iteration of the transformation process. Using this method, stable control parameters were also found for the wobble chair (Figure 4b), a feat that we had not yet achieved using the direct method. The controller gains for the wobble chair were G_p = 3x10^5 Nm/rad and G_d = 200 Nm/rad/s.

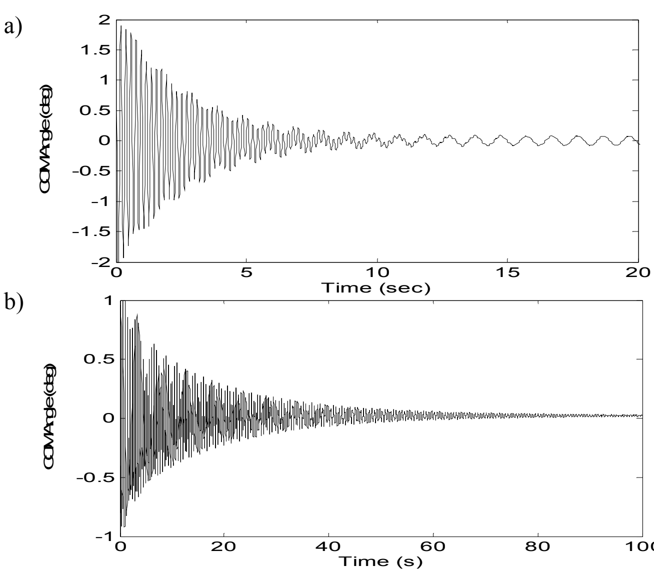

Figure 4: Convergence of the combined center of mass towards the upright vertical position for the Acrobot (a) and the wobble chair (b).

In addition, an analysis was conducted to test for stability of these two systems over a large range of proportional and derivative gain parameters. A two dimensional representation of G_p verses G_d makes up the control space ($\Re^2$). A direct method was used to evaluate the stability of the system for various combinations of G_p and G_d. An initial condition of θ_1 = -2.2 and θ_2 = 1.8 was used to begin each simulation. This initial condition is near, but slightly off an equilibrium configuration. The trajectory was tracked as it evolved under the influence of the controller. If the trajectory left the neighborhood of the equilibrium manifold (>5°), it was deemed unstable; otherwise, it was considered stable. The results showed the Acrobot to have a larger area of control space exhibiting stable behavior than that which was present for the wobble chair.

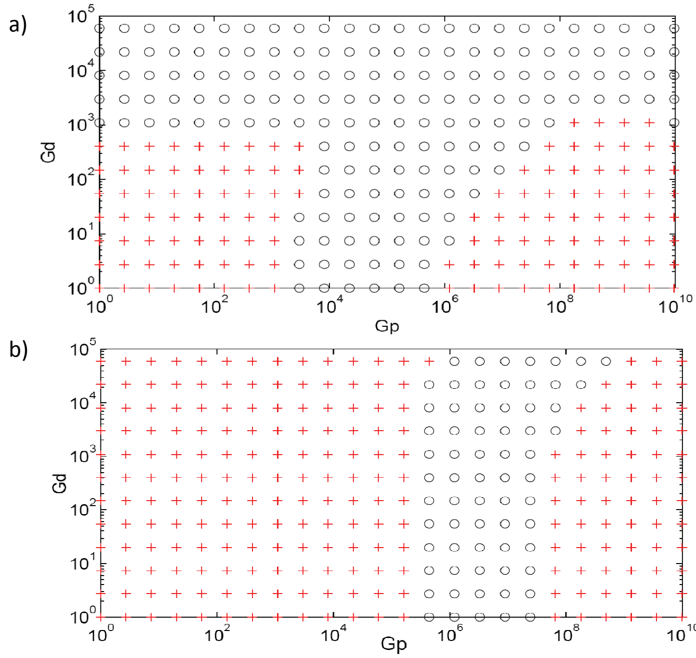

Figure 5: The control space for the Acrobot (a) and wobble chair (b). Controller gains that resulted in stable behavior were indicated with a "o" and unstable behavior was marked as a "+".

DISCUSSION

A method was presented that utilized transformations to make a difficult problem easier to solve. Recognizing that the Acrobot was topologically equivalent to the wobble chair enabled us to ultimately find a solution for a difficult problem that we were having trouble solving directly. The transformation methods used in this research are not specific to torso stability analysis and have broader application beyond the field of biomechanics.

ACKNOWLEDGEMENTS

We would like to thank Dr. Craig Woolsey for introducing us to the Acrobot. In addition, support for this work was provided by the Kevin P. Granata Occupational Safety and Health Pilot Research Program of Virginia Tech's Center for Innovation in Construction Safety and Health.

REFERENCES

1. Tanaka, M.L., S.D. Ross, and M.A. Nussbaum, *Mathematical modeling and simulation of seated stability.* J Biomech, 2009.
2. Murray, R. and J. Hauser, *A case study in approximate linearization: The Acrobot Example*, Proc. American Control Conference, 1990.