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A CONTINUOUS METHOD TO QUANTIFY STRESS-STRAIN BEHAVIOR OF BIOLOGIC MATERIALS

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Introduction

Mathematical models of biologic structures such as the knee, spine, and hand provide opportunities to better understand complex interactions and to simulate treatment outcomes. Accurate estimates of the material properties of biologic soft tissues are critical to the fidelity of biomechanical models. Biologic soft tissues have a nonlinear mechanical behavior characterized by an exponential toe region, described with $\sigma = A\left(e^{B\varepsilon} - 1\right)$ followed by a linear elastic region [1]. This has been used to describe the stress-strain properties of many soft tissues [2-4]. Using these traditional models, the two curves are independently fit to a separate set of data points. Thus, it is possible that the two portions of the modeled stress-strain curves do not necessarily exhibit both C^0 and C^1 continuity, and would not accurately represent a real material.

Herein, a "Continuous Method" is presented that enforces continuity in both the stress-strain curve and its derivative by optimizing the curve fit for both regions simultaneously. Performance of this method is compared to the traditional "Piecewise Method".

Methods

Both the traditional Piecewise Method and the Continuous Method were implemented via custom MATLAB® (The MathWorks, Natick, MA) code. To demonstrate effectiveness, both methods were used to analyze four data sets: ideal data, noisy idealized data produced by adding random noise to the idealized data (moderate and high levels) and measured data. Results for the ideal data and measured data are shown in this abstract.

Piecewise Method -- The Piecewise Method employs a linear curve fit to the high-strain end of the stress-strain data. Data points are included in the linear region until the R^2 of the fitted line dips below an *a priori* determined threshold value (R^2_{cutoff}= 0.99 in this work) [5]. The remaining lower strain points are fit to an exponential curve as described above [3].

Continuous Method -- A mathematical model was developed that contains an exponential region and a linear region of the stress-strain curve. To maintain a continuous elastic modulus *E*, the slope of the linear portion was defined as the slope of the stress-stain curve at the transition point *(p, q)* between the exponential and linear regions. The stress-strain function is thus defined by,

$$\sigma = \begin{cases} A\left(e^{B\varepsilon} - 1\right) & \forall \varepsilon \le p \\ E_{(p,q)}\left(\varepsilon - p\right) + q & \forall \varepsilon > p \end{cases}$$

where *E(p, q)* is the elastic modulus at point *(p, q)*.

Custom code was developed using MATLAB® software to compute the optimal parameters values for *A*, *B*, and *p* by minimizing the mean square error (*MSE*). This yields the least-squares error between the modeled function and the experimental data. The optimization is performed on the entire curve, *simultaneously fitting the exponential section and linear section*, and that the location of the transition point, *p*, is a continuous variable included in the optimization process.

Data Sets -- The ideal data set was generated using an exponential curve that smoothly transitioned to a line,

$$\sigma = \begin{cases} 0.200\left(e^{35.0\varepsilon} - 1\right) & \forall \varepsilon \le 0.10 \\ 232\left(\varepsilon - 0.10\right) + 6.42 & \forall \varepsilon > 0.10 \end{cases}$$

The curve had C^0 and C^1 continuity and a known transition point at (0.10, 6.42). The measured data set was collected from a sample of porcine lateral meniscus using previously-developed methods for tensile testing [4].

Results

Ideal Data -- The Piecewise Method generated a stress-strain curve with a discontinuity of 3.03 MPa at the transition point, or 319% of the actual stress magnitude at this location (Figure 1). The discontinuity in its derivative resulted in a modulus increase from 25.7 MPa to 206 MPa at the transition. The estimates for parameters A and B had errors of 374% and 60.9%, respectively (Table 1). However, the elastic modulus was estimated with an error of 11.2%. The Piecewise method was inaccurate at estimating the location of the actual transition point, underestimating the strain by 50% and the stress by 109%.

In contrast, the Continuous Method generated an unbroken curve through the transition point (Figure 1) and without discontinuity in modulus. The parameters A and B were estimated within 7.00% and 2.57% of the known values, and the elastic modulus, *E*, within 0.431%. The location of the transition point was within 1.10% of the known strain and 1.87% of the known stress. Thus, the Continuous Method performed better than the Piecewise Method with ideal data.

Measured Data -- Application of the Piecewise Method resulted in a discontinuity of 1.9 MPa at the transition. The model closely approximated the measured data in the exponential region, but it deviated from the actual measured data points in the linear region (Figure 2). The Continuous Method yielded a curve that more closely tracks the measured data points and had a MSE over 20 times lower that obtained using the Piecewise method (Table 2).

Discussion and Conclusions

The Continuous Method dramatically reduced errors in estimating the parameters *A*, *B*, *E*, *p*, and *q* when applied to ideal data with known solutions. When applied to measured data without a known solution, the Continuous Method had a much better fit. The Piecewise Method yielded discontinuities in both the stress strain curves and the modulus, a known shortcoming of this method. In contrast, the Continuous Method eliminated both C^0 and C^1 discontinuities, by design, resulting in a continuous curve and modulus.

The Continuous Method may be a more accurate way to estimate material parameters of soft biological tissues. *A key element of the Continuous Method approach is to simultaneously solve a problem that had historically been solved sequentially.* The use of this simultaneous approach may broadly impact the fields of engineering modeling and biomechanics.

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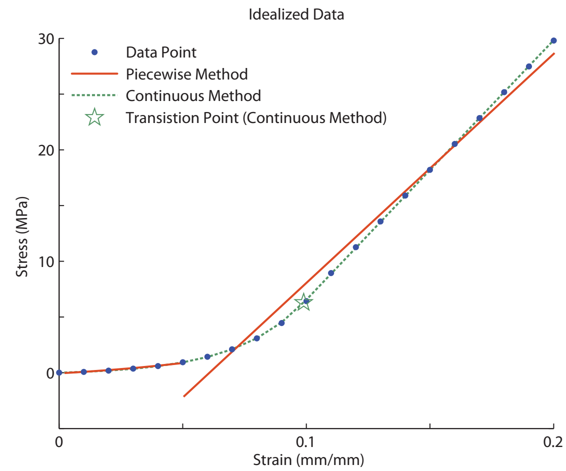

Figure 1: Ideal Data Set

Table 1: Ideal Data Results (Percent Error).

	A (MPa)	B	E (MPa)	p	q (MPa)	MSE
Known	0.2	35	232	0.1	6.42	
Piece. Meth.	0.947 (374%)	13.7 (61%)	206 (11%)	0.05 (50%)	-0.585 (109%)	1.09
Cont. Meth.	0.186 (7.0%)	35.9 (2.6%)	233 (0.4%)	0.099 (1.1%)	6.3 (1.9%)	0.0015

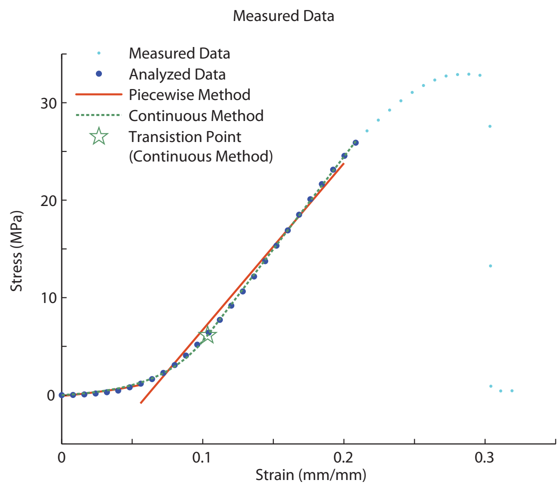

Figure 2: Measured Data Set

Table 2: Measured Data Results

	A (MPa)	B	E (MPa)	p	q (MPa)	MSE
Piece. Meth.	0.882	14.8	171	0.0561	0.0419	0.46
Cont. Meth.	0.307	29.4	190	0.103	6.14	0.021