

# Stability Regions Observed in Controlled Random Walks

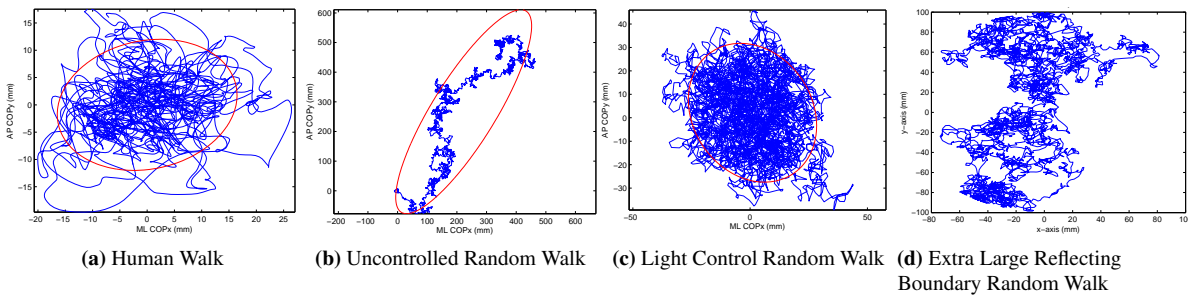
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## INTRODUCTION

Human postural stability remains a major concern in injury prevention. Human posture by nature is inherently unstable. Despite this, human postural sway demonstrates two regions of stability. For the past twenty years, researchers have believed that these regions exist as the result of both an open and closed loop control [1, 2]. The short and long term components are attributed to spinal cord reflexes and cognitive time delay respectively. Alternatively, the short and long term components could simply be the nature of a controlled system. This posed the question: Will any level of control applied to a system cause two regions of stability? Our research was conducted to ascertain if applying a simple controller to a random walk will demonstrate two regions of stability.

## METHODS

Using MATLAB, we created two dimensional random walk simulations with varying levels of control. Starting at the origin, the simulation was generated by moving a fixed distance in a random direction with each time step. A simple proportional controller was added to the random walk which displaced the point in the direction of the origin proportional to its distance. Four levels of control were applied to the random walk: no control, light control, moderate control, and strong control (Figure 1).



**Figure 1.** Postural Stability

A circular boundary with a reflecting surface was also implemented as a simple form of control. The strength of the controller was determined by the radius of the circle boundary. Four strengths were applied to the random walk: extra-large (100m), large (60m), medium (20m), and small (6m). If the projected point exceeded the circular boundary, it was reflected back into the circle (Figure 2). After locating the point of intersection,  $(x_p, y_p)$ , between the walk trajectory and the boundary, the angle of incidence  $\theta$  is found using the Law of Cosines and inverse trigonometric functions. The coordinates for the new reflected point,  $(x_2, y_2)$ , are given by  $x_2 = c \cos(B + \rho)$  and  $y_2 = c \sin(B + \rho)$  (Figure 2).

Stability diffusion analysis was applied to simulation data sets to determine whether two regions of stability were produced. The stability diffusion coefficients and scaling components were calculated using established methods [1] and are explained below. The relationship between the change in the distance and the evolution time is given by,  $\langle \Delta x^2 \rangle = 2D\Delta t$  where  $\Delta x$  is the distance traveled from the initial starting point,  $\Delta t$  is the trajectory evolution time, and  $D$  is the diffusion coefficient used to quantify the rate of dispersion [3].

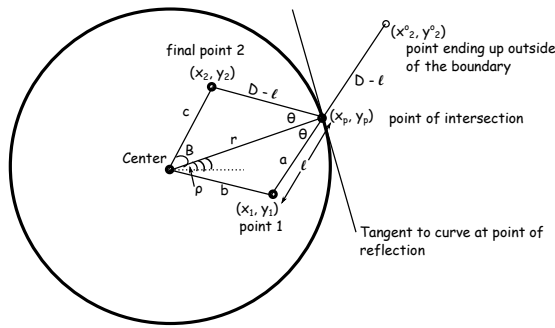


Figure 2. Reflecting Boundary

Movement was divided into two time regions with the separation between short term and long term behavior determined by the first peak in the  $\Delta x^2$  (Figure 3a). A linear curve fit was applied to the short and long term components. Each region was analyzed separately. Stability diffusion analysis was applied to human postural sway data to verify our method and provide a reference for comparison to the random walk data.

RESULTS

As expected, the human postural sway data demonstrated two regions of stability (Figure 3a). Similarly, the two dimensional random walk produced both regions of stability when any level of control was implemented (Figure 3b). Increasing the strength of the controller caused the second region of stability to develop sooner. In contrast, only one region of stability was exhibited when the two dimensional random walk without a controller was tested. The bounded system showed two regions of stability at small and medium radii length (Figure 3c). The large boundary system displayed a second region after an extended period of evolution. The system with the farthest boundary, extra-large, appeared to have one region of stability. However, a second region became observable after a prolonged evolution time.

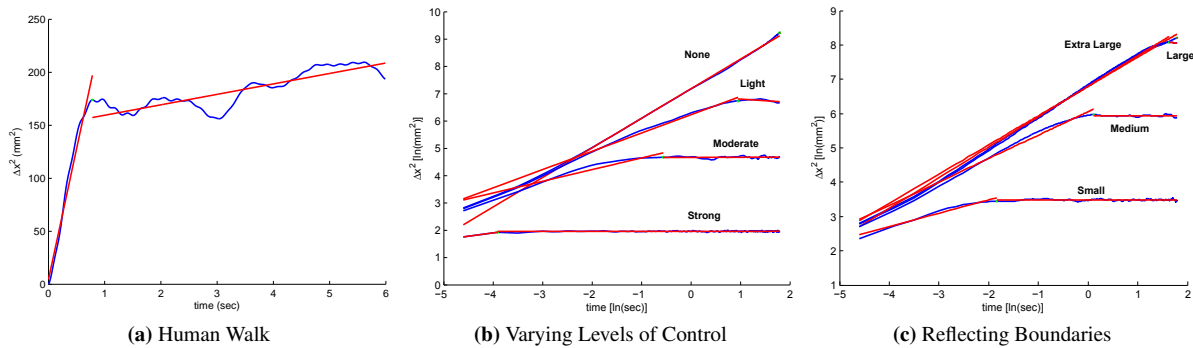


Figure 3. Stability Diffusion

DISCUSSION

Based on our findings, it appears that any level of control applied to a random walk will produce two regions of stability. Further studies will help to elucidate whether these regions of stability are a result of human factors or the consequence of a controlled system.

References

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