

DETECTING DYNAMICAL BOUNDARIES FROM KINEMATIC DATA IN BIOMECHANICS

Shane D. Ross

Engineering Science and Mechanics
Virginia Tech
Blacksburg, VA 24061, USA
sdross@vt.edu

Martin L. Tanaka

Engineering and Technology
Western Carolina University
Cullowhee, NC 28723, USA
mtanaka@wcu.edu

Carmine Senatore

Engineering Science and Mechanics
Virginia Tech
Blacksburg, VA 24061, USA
themast@vt.edu

ABSTRACT

Ridges in the state space distribution of finite-time Lyapunov exponents can be used to locate dynamical boundaries. We describe a method for obtaining dynamical boundaries using only trajectories reconstructed from time series, expanding on the current approach which requires a vector field in the state space. We analyze problems in musculoskeletal biomechanics, considered as exemplars of a class of experimental systems that contain separatrix features. Particular focus is given to postural control and balance, considering both models and experimental data. Our success in determining the boundary between recovery and failure in human balance activities suggests this approach will provide new robust stability measures, as well as measures of fall risk, that currently are not available and may have benefits for the analysis and prevention of low back pain and falls leading to injury, both of which affect a significant portion of the population.

INTRODUCTION

The identification of frontiers between qualitatively different kinds of behavior in a dynamical system is important in many applications. Increasingly, systems of interest are determined not by analytically defined model systems, but by noisy time series data sets resulting from experiments or complex simulations. Recently, the concept of Lagrangian coherent structures, ridges in the state space distribution of finite-time Lyapunov exponents (FTLEs), has been used to locate these dynamical boundaries. The current approach to finding these structures requires a vector field in the state space at each instant in time. In this paper, we demonstrate a method that uses only trajectories reconstructed from time series. To demonstrate this approach, we analyze problems in musculoskeletal biomechanics, considered as exemplars of a class of experimental systems that contain boundaries which

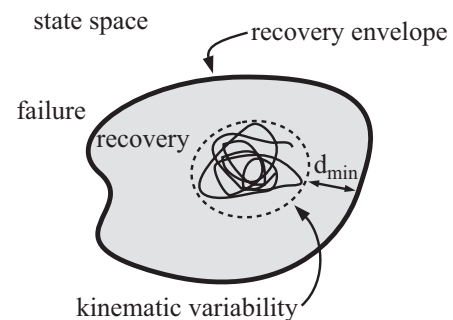


FIGURE 1. Shown schematically, the boundary between the states corresponding to safe motion and failure is given by a recovery envelope (thick line). The minimum state space distance between this envelope and the subject's kinematic variability can measure their risk of injury.

have gone largely ignored, despite their importance for determining possible behaviors and the relationship between them.

RECOVERY ENVELOPE

We focus specifically on the study of human balance control while sitting, standing or walking. We demonstrate the possibility of determining a 'threshold of stability' or recovery envelope—the set of states separating balanced states from falling states, that is, the boundary of a basin of stability—from experimental time series data, see Fig. 1 [1, 2, 3]. Specifically, we find separatrices in time series data as ridges of high sensitivity (high FTLE). In high dimension, a ridge corresponds to a codimension one manifold in state space. Previous research on time series analysis has focused on determining whether the dynamics come from chaotic deterministic processes or noise [4, 5, 6]. But

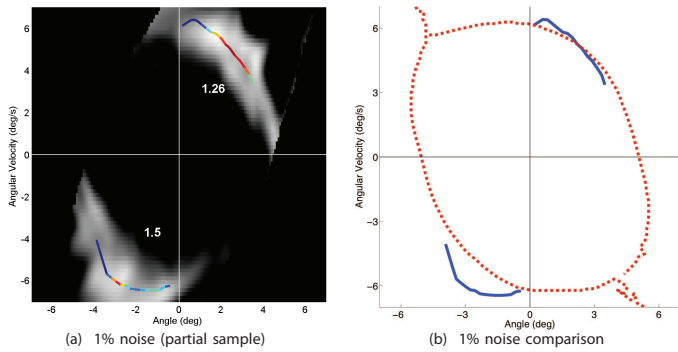


FIGURE 2. (a) A portion of a dynamical boundary found by analyzing the FTLE field resulting from the trajectories of a single degree of freedom system. Only portions of the complete boundary are accessible by this method. The flux for the upper-right and lower-left portions are also shown. (b) A comparison of the dynamical boundaries from the partial sample (solid) and full-grid data.

there is more information to be found in the data—namely the structure and interconnectedness of the state space as determined by boundaries between qualitatively different kinds of behavior.

DYNAMICAL BOUNDARIES FROM TRAJECTORIES

We have developed a method to construct the FTLE field using only trajectory data [1]. This approach eliminates the need for the generating vector field, which is generally unavailable in a biomechanics environment. Typically in biomechanics experiments a finite number of body segments are tracked and stored as time series data. By properly performing state space reconstruction, the FTLE field is estimated as the rate of separation of neighboring trajectories. Once the FTLE field is available, we use a systematic method for not only extracting—but also characterizing—dynamical boundaries which is useful for tracking and identifying individual features that may merit further analysis.

POSTURAL STABILITY REDUCED ORDER MODELS

We consider two models for postural stability, with particular attention to modeling the wobble chair, an experimental apparatus used to obtain postural data. One and two degree of freedom models have been developed and their dynamical boundaries analyzed using the novel extraction and characterization procedure.

As in an experimental setting, we consider the effect of sparse data (having only a few trials) on the determination of the recovery envelope. Unlike full sampling, which is possible from simulated data, the data we consider from a few simulated trials only contains information about the portion of state space explored by the trials (Fig. 2(a)). Despite being less complete than boundaries generated from full grid data, the portion of the boundary which is sampled is in the same general location, as shown by the comparison in Fig. 2(b). This numerical experiment from a single degree of freedom model provides confidence that partial sampling of the state space using experimental trials

can be used to detect boundaries, as well as estimate the differential and average flux across it.

We also consider a more realistic two degree of freedom model, in which the FTLE field was generated using forward dynamics simulations for a fixed (and shot) horizon time based on a regular grid of initial conditions. In this case, the dynamical boundary is a three-dimensional surface separating the four-dimensional state space into two distinct regions, stable and unstable. We verified that the dynamical boundary obtained from relatively short time sensitivity analysis actually bounds the basin of stability by comparing with a ‘brute-force’ measure of stability.

CONCLUSIONS

The extension of separatrix methods to experimental time series analysis of mechanical and biomechanical systems provides better means of comparison with analytical models and gives greater insight into possible—but as yet unobserved—behaviors, based on the determined state space structure and interconnectivity. Furthermore, detection of separatrices dividing regions of failure from regions of safety will provide new robust stability measures that provide additional information over what is currently available. The methods described apply to other areas where nonlinear time series analysis is used to analyze not only mechanical systems, but also non-mechanical systems, such as meteorological, financial, psychological, and population observations. Whereas previous research in time series analysis using finite-time Lyapunov exponents has focused only on determining whether the dynamics come from deterministic chaos or noise, we have shown that there may be more information to be mined from the data. Structure within the underlying deterministic system may yield information about state-space transport phenomena that is critical to understanding the behavior of the system.

REFERENCES

- [1] Tanaka, M. L., and Ross, S. D., 2009. “Separatrices and basins of stability from time series data: an application to biodynamics”. *Nonlinear Dynamics*, **58**(1-2), pp. 1–21.
- [2] Tanaka, M. L., Nussbaum, M. A., and Ross, S. D., 2009. “Evaluation of the threshold of stability for the human spine”. *Journal of Biomechanics*, **42**(8), pp. 1017 – 1022.
- [3] Ross, S. D., Tanaka, M. L., and Senatore, C., 2010. “Detecting dynamical boundaries from kinematic data in biomechanics”. *Chaos*, **20**, p. 017507.
- [4] Dingwell, J. B., 2006. “Lyapunov exponent”. In *The Wiley Encyclopedia of Biomedical Engineering*, M. Akay, ed. Wiley, New York.
- [5] Falconer, I., Gottwald, G. A., Melbourne, I., and Wormnes, K., 2007. “Application of the 0-1 test for chaos to experimental data”. *SIAM Journal on Applied Dynamical Systems*, **6**(2), pp. 395–402.
- [6] Poon, C.-S., and Barahona, M., 2001. “Titration of chaos with added noise”. *Proceedings of the National Academy of Sciences*, **98**, pp. 7107–7112.